

Net-proton probability distribution in heavy ion collisions

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We compute net-proton probability distributions in heavy ion collisions within the hadron resonance gas model. The model results are compared with data taken by the STAR Collaboration in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV for different centralities. We show that in peripheral Au-Au collisions the measured distributions, and the resulting first four moments of net-proton fluctuations, are consistent with results obtained from the hadron resonance gas model. However, data taken in central Au-Au collisions differ from the predictions of the model. The observed deviations can not be attributed to uncertainties in model parameters. We discuss possible interpretations of the observed deviations.

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I. INTRODUCTION

One of the objectives of heavy ion experiments at CERN and BNL is to probe properties of the QCD phase diagram related to deconfinement and chiral symmetry restoration. To experimentally verify the phase change in a medium created in such collisions, one needs observables that are sensitive to critical behavior. Modifications in the magnitude of fluctuations or in the corresponding susceptibilities of conserved charges have been suggested as possible signals for chiral symmetry restoration and deconfinement [1–6].

Fluctuations of baryon number and electric charge diverge at the hypothetical critical end point in the QCD phase diagram at non-zero temperature and baryon chemical potential, while they remain finite along the cross-over boundary. Consequently, large fluctuations of baryon number and electric charge as well as a non monotonic behavior of these fluctuations as function of the collision energy in heavy ion collisions have been proposed as a signature for the QCD critical end point [1, 3, 6].

It also has been argued that even in the absence of a critical end point, fluctuations of conserved charges, and their higher order cumulants, can be used to identify the phase boundary. It is expected that the fluctuations are modified if the chemical freeze-out, i.e., the generation of hadrons and their fluctuations, occurs shortly after the system passed through a region where quarks were deconfined and chiral symmetry was partially restored [5, 7, 8]. Thus, fluctuations in the hadronic phase can show structures attributed to critical dynamics at the restoration of chiral symmetry.

The critical region characterizing the cross-over transition in the QCD phase diagram is expected to be located close to the freeze-out curve extracted from heavy ion experiments [9]. On this phenomenologically determined

curve all particle yields achieve their measured values [10–13]. Thermodynamics at freeze-out is, to a first approximation, well described by the hadron resonance gas (HRG) model, which was shown to be very successful in describing not only the data on particle yields but also the thermodynamics of a strongly interacting medium at low temperature as computed in lattice QCD [2, 14–18].

If chemical freeze-out occurs near or at the QCD phase boundary, this should be reflected in the higher order cumulants of charge fluctuations since the sensitivity to critical dynamics grows with increasing order [7]. Consequently, the values of higher order cumulants can differ significantly from the results of the HRG along the freeze-out curve even if lower order cumulants agree. In particular, at vanishing chemical potential, the sixth and higher order cumulants can even be negative in the hadronic phase while the HRG yields positive values everywhere [7]. In fact, the results of the HRG model on cumulants of charge fluctuations can serve as a theoretical baseline for the analysis of heavy ion collisions [2, 7, 14]. In equilibrium, any deviation from the HRG model would be a reflection of genuine QCD properties not accounted for by the model and could constitute evidence for critical phenomena at the time of hadronization.

Recently first data on charge fluctuations and higher order cumulants, identified through net-proton fluctuations, were obtained by the STAR Collaboration in Au-Au collisions at several collision energies [19, 20]. To explore possible signs of criticality, the STAR data on the first four cumulants were compared to HRG [7, 20] and lattice QCD [21, 22] results. The basic properties of the measured fluctuations and ratios of cumulants are consistent with the expectations based on HRG as well as on lattice QCD calculations. However, a more detailed comparison of the HRG model with STAR data reveals that at high energies, deviations can not be excluded [7, 20].

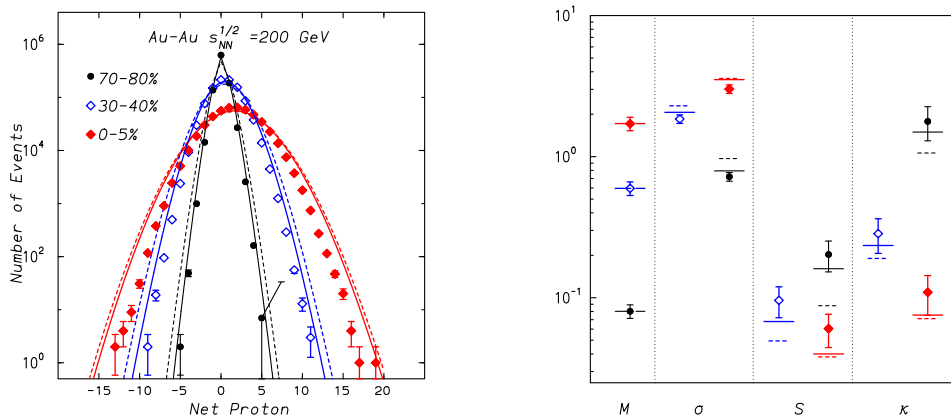


FIG. 1: Left-hand figure: Net-proton distributions calculated in the hadron resonance gas using Eq. 4 and compared with STAR data for Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV for different centralities [19]. The full-lines are obtained with experimental inputs for proton $\langle N_p \rangle$ and anti-protons $\langle N_{\bar{p}} \rangle$ yields, while the broken-lines are obtained with $\langle N_p \rangle$ and $\langle N_{\bar{p}} \rangle$ computed in the thermal model with the parameters at chemical freeze-out taken from Ref. [13]. Right-hand figure: Mean (M), variance (σ), skewness (S) and kurtosis (κ) calculated from the probability distributions shown in Fig. 1 (left) for different centralities. Data are from STAR [19].

All moments of net-proton fluctuations as well as the related cumulants can be calculated once the underlying probability distribution is known. Therefore, it is interesting to confront the net-proton distributions obtained for the HRG with those measured in heavy ion collisions. Since the probability distributions contain information on all cumulants, such a comparison may provide useful insights into the origin of possible deviations from the HRG baseline. An analysis of this kind may also provide additional information on the relation between chemical freeze-out and the QCD cross-over transition or even on the existence of a critical end point in the QCD phase diagram.

In the following we calculate the net-proton probability distributions in the HRG model. We show that in this model the net-proton distribution can be expressed solely in terms of the measurable yields of protons and anti-protons. The HRG model results are compared with data taken by the STAR Collaboration in Au-Au collisions at RHIC at $\sqrt{s_{NN}} = 200$ GeV. For central collisions at this energy, the HRG model yields a distribution which is broader than the one seen experimentally. We discuss the possible origin of the deviations and point out that they are in qualitative agreement with the differences between HRG model calculations and (lattice) QCD results found in the vicinity of the QCD phase boundary and above.

II. PROBABILITY DISTRIBUTION OF CONSERVED CHARGES

Consider a sub-volume V of a thermodynamic system described by the grand canonical ensemble consisting of charged particles q and anti-particles \bar{q} at a given temperature T and chemical potential μ . The latter is related to the conserved net charge $N = N_q - N_{\bar{q}}$. The prob-

ability distribution $P(N)$ for finding a net charge number N in the volume V is given in terms of the canonical $Z(T, V, N)$ and grand canonical $\mathcal{Z}(T, V, \mu)$ partition functions [10, 23, 24],

$$P(N) = Z(T, V, N) e^{\hat{\mu}N - V\beta p(T, \mu)}. \quad (1)$$

Here we have introduced the thermodynamic pressure, $\ln \mathcal{Z} = V\beta p(T, \mu)$, as well as the shorthand notation $\hat{\mu} = \beta\mu$ and $\beta = 1/T$.

The canonical partition function $Z(T, V, N)$ can be obtained from the thermodynamic pressure through the discrete Fourier transform

$$Z(T, V, N) = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-i\phi N} e^{\beta V p(T, i\phi/\beta)}, \quad (2)$$

where the chemical potential was Wick rotated by the substitution $\hat{\mu} \rightarrow i\phi$. Eqs. (1) and (2) define the probability distribution of a conserved charge in a sub-volume V of a thermal system described by the thermodynamic pressure $p(T, \mu)$.

In the following we focus on fluctuations of the net-baryon number and consider the corresponding probability distribution in a strongly interacting medium. We model the thermodynamics of this system using the HRG partition function, which contains all relevant degrees of freedom in the hadronic phase and implicitly includes the interactions responsible for resonance formation. In this model the thermodynamic pressure is a sum of meson and baryon contributions. This implies that only the baryonic pressure p_B contributes to the probability distribution of the net-baryon number. In the HRG model p_B consists of contributions from all baryons and baryonic resonances [10]. In the Boltzmann approximation, $\beta V p_B(T, \mu) = b(T, V, \mu) + \bar{b}(T, V, \mu)$, where b and \bar{b} is the

mean number of baryons and anti-baryons respectively,

$$b(T, V, \mu) = \frac{VT}{2\pi^2} \sum_{i \in \text{baryons}} g_i m_i^2 K_2(\beta m_i) e^{\hat{\mu} + \beta \vec{q}_i \vec{\mu}}, \quad (3)$$

Here $\vec{q}_i = (S_i, Q_i)$ is a two-component vector composed of the strangeness and electric charge carried by particle i , $\vec{\mu}_q = (\mu_S, \mu_Q)$ the corresponding chemical potential vector, g_i the spin-isospin degeneracy factor and K_2 is a modified Bessel function. The mean number of anti-baryons \bar{b} is obtained by the substitution $\mu \rightarrow -\mu$ for all relevant chemical potentials.

In the hadron resonance gas model, the canonical partition function $Z(T, V, N)$ is computed directly from Eq. (2), using the thermodynamic pressure discussed above [10]. The resulting probability distribution of net-baryon number can be expressed solely in terms of the mean number of baryons and anti-baryons,

$$P(N) = \left(\frac{b}{\bar{b}}\right)^{N/2} I_N(2\sqrt{b\bar{b}}) \exp[-(b + \bar{b})], \quad (4)$$

where $I_N(x)$ is a modified Bessel function.

We note that the above arguments remain valid also for subsystems which are limited not only in position space, but more generally in phase space. In particular, the introduction of cuts in momentum space leave Eqs. (1), (2) and (4) unchanged after an appropriate redefinition of the partition functions and densities. Moreover, the restriction to one particle species, e.g. protons with proper account for resonance decays, is easily accommodated.

III. PROBABILITY DISTRIBUTION OF THE NET-PROTON NUMBER

In this section we confront the HRG model results for the probability distribution of the net-proton number with data of the STAR Collaboration [19, 20]. The data is obtained at mid-rapidity in a restricted range of transverse momentum, $0.4 \text{ GeV} \leq p_T \leq 0.8 \text{ GeV}$. The probability distribution for protons is readily obtained from Eq. (4), by replacing the mean number of baryons b and anti-baryons \bar{b} by that of protons $\langle N_p \rangle$ and anti-protons $\langle N_{\bar{p}} \rangle$, respectively.

Using Eq. (4) we can compute the net-proton distribution provided we have access to the mean values $\langle N_p \rangle$ and $\langle N_{\bar{p}} \rangle$ measured in the same kinematic window. For Au-Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$, STAR data on the p_t -distribution of protons and anti-protons is available at several centralities [13]. By integrating the p_t -spectra of anti-protons in the p_t -window where the net-proton number was obtained, we find: $\langle N_{\bar{p}} \rangle = 5.233(95)$, $\langle N_{\bar{p}} \rangle = 1.838(7)$ and $\langle N_{\bar{p}} \rangle = 2.844(98)$ for (0-5%)-central, (30-40%)-mid-central and (70-80%)-peripheral collisions, respectively. Since the proton data were measured in a slightly larger p_t window, we avoid systematic errors that may arise from an extrapolation by computing $\langle N_p \rangle$

from the net-proton yields ($M = \langle N_p \rangle - \langle N_{\bar{p}} \rangle$), with $M \simeq 1.715$, $M \simeq 0.597$ and $M \simeq 0.08$ for central, mid-central and peripheral collisions [19].

In Fig. 1 (left) we compare the STAR data on the net-proton multiplicity distribution in Au+Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$ [19, 20] with the probability distributions obtained in the HRG model (Eq. (4)), using the experimental data on M and $\langle N_{\bar{p}} \rangle$ as input. The data correspond to several centrality bins in the rapidity window $|y| < 0.5$. The distribution in Eq. (4) is normalized to unity. In order to confront the HRG model with data on an absolute scale, we adjust the normalization to that of the experimental data in each centrality bin.

Fig. 1 (left) shows that in peripheral collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$ the HRG model results agree well with the data. However, with increasing centrality deviations develop; in central collisions the hadron resonance gas yields a distribution, which is broader than the experimental one. Below we argue that such deviations are expected if the freeze-out conditions probed by fluctuations are located close to the QCD cross-over temperature. Consequently, the deviations observed in Fig. 1 (left) and the dependence on centrality, could be an indication for critical behavior.

We have also calculated the number of protons and anti-protons with chemical freeze-out parameters adjusted to the multiplicities measured in Au-Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$. The centrality dependence of the freeze-out parameters were determined in Ref. [13]. In this approach an additional input parameter is required, the effective volume (V), which we fix by requiring that the measured net-proton number M at chemical freeze-out is reproduced by the HRG model. As shown in Fig. 1 (left) the probability distributions obtained in this way are consistent with those computed directly from the measured yields of protons and anti-protons using Eq. (4). The consistency of the two approaches strengthens our conclusion that the HRG model does not describe the net-proton probability distribution $P(N)$ for central Au-Au collision at $\sqrt{s_{NN}} = 200 \text{ GeV}$.

Recently the STAR Collaboration also presented preliminary data on the net-proton distribution in Au-Au collisions at $\sqrt{s_{NN}} = 39 \text{ GeV}$ for various centralities [20]. It would be interesting to compare this data with the HRG model. In this case, however, the corresponding data on p_t -distributions, which would allow one to determine $\langle N_p \rangle$ and $\langle N_{\bar{p}} \rangle$ directly from the experiment is not available. Thus, here we can only follow the second approach, i.e., employ the measured net-proton number M and the chemical freeze-out parameters from Ref. [12] to determine $\langle N_p \rangle$ and $\langle N_{\bar{p}} \rangle$, which are then used as input for the calculation of $P(N)$ based on Eq. (4).

A first comparison of the HRG model calculation with the probability distribution obtained for central Au-Au collisions at $\sqrt{s_{NN}} = 39 \text{ GeV}$ indicates that at this energy the shape and magnitude of the measured net-proton distribution, and consequently also the first four measured moments, are described well by the HRG

model. This suggests that in central Au-Au collisions at $\sqrt{s_{NN}} = 39$ GeV the fluctuations as well as the particle yields are characterized by the thermodynamic freeze-out conditions corresponding to the statistical operator of the hadron resonance gas model. Clearly, this result needs to be confirmed by the final STAR data at this lower energy. If correct, this would suggest that the deviations from the HRG results grow with increasing energy. This feature can also be tested at the LHC.

The observed deviations in the probability distribution are also manifested in differences between calculated and measured cumulants of the net-proton fluctuations. In Fig. 1 (right) we show the mean, variance, skewness and kurtosis obtained from the probability distributions in Fig. 1 (left). The HRG model, with experimental input for proton and anti-proton yields a slightly better description of all four moments. However, a good overall description is obtained only for peripheral collisions.

The fact, that the HRG model yields a distribution which is broader than the experimental one implies that deviations arise already on the level of the second order cumulant (variance), which has the smallest experimental error. This is expected if the particle freeze-out occurs near the QCD cross-over transition. In the cross-over region the baryon number susceptibility (χ_2^B), i.e., the second order cumulant $\sigma^2 = VT^3\chi_2^B$ [7], keeps rising steeply with temperature in the HRG model, while in QCD calculations it bends over and eventually approaches a finite value at high temperatures. In the Gaussian approximation (the leading order cumulant expansion) to the probability distribution, $P(N) \sim \exp[-N^2/(2\sigma^2)]$, this implies that the distribution in QCD is narrower than in the HRG model.

A possible interpretation of this effect may be related to the proximity of the freeze-out and cross-over regions probed at the highest beam energy. Lattice calculations

suggest that at larger μ , the freeze-out curve the cross-over transition separate [4, 25]. Hence, one expects that the fluctuations reflect the critical dynamics at the cross-over transition only at the highest beam energies.

IV. CONCLUSIONS

We have analyzed properties of the net-proton probability distributions in heavy ion collisions within the hadron resonance gas model. In this model these distributions can be expressed solely in terms of the mean numbers of protons and anti-protons in a thermal system. This provides a direct and unambiguous way to compare experimental data with model predictions.

We have shown that the HRG model describes the data obtained by STAR in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV for peripheral events but differ for central ones. Since the probability distributions are computed directly from the measured proton and anti-proton yields, this deviation is not due to uncertainties in model parameters. We suggest that this effect could be due to the proximity of the freeze-out and cross-over regions at the highest beam energy. In order to substantiate this interpretation, data on the net-proton distribution at lower RHIC energies and at LHC energies are needed.

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